Chemical Evolution of the Universe
Problem sheet 9

1. Let $m = \frac{M}{M_\odot}$. Consider the Salpeter Initial Mass Function (IMF), given by $\Phi(m) = A\, m^{-2.35}$, where A is a constant, with mass limits $m_l = 0.1$ and $m_u = 100$.
   (a) Compute A from the standard normalisation condition $\int_{m_l}^{m_u} m\, \Phi(m)\, dm = 1$.
   (b) Calculate the median mass by number, $m_{N,1/2}$, defined by $f_N(m_{N,1/2}) = \frac{1}{2}$.
   (c) Calculate the fraction of mass contained in stars with $m > m_{N,1/2}$, i.e. $f_M(m_{N,1/2})$.
   (d) Now compute the median mass by mass, $m_{M,1/2}$, defined by $f_M(m_{M,1/2}) = \frac{1}{2}$.
   (e) Consider a galaxy with a constant star formation rate of $\psi = 20\, M_\odot/\text{yr}$. Assuming that every star with $m > 8$ explodes as a supernova (SN), how many years do we have to wait (on average) in order to see one SN in this galaxy?

6 points

2. Compute the lock-up and return fractions, α and R, for Salpeter's IMF (see above) in the instantaneous recycling approximation, assuming $m = 1$ to be the boundary between stars that have already died and those that have not.

2 points

3. Consider a stellar cluster of age t_C and some star formation history $\psi(t)$. Let us assume that we know the distance to this cluster and that we have hence been able to measure the present-day mass function of its main sequence stars, $N(M)$. How can these data be used to constrain the unknown IMF of the cluster, $\Phi(M)$ (assumed to be constant as a function of time)?
   (a) Assume that $\tau(M) \gg t_C$ for all $M < M_1$, where $\tau(M)$ is the main sequence lifetime of a star of mass M. What is the relationship between $N(M)$ and $\Phi(M)$ for $M < M_1$?
   (b) Further assume that $\tau(M) \ll t_C$ for $M > M_2$ and that $\psi(t)$ is approximately constant over the small time interval $[t_C - \tau(M_2), t_C]$. What is the relationship between $N(M)$ and $\Phi(M)$ for $M > M_2$?
   (c) Further assume a linear relationship between $\log \Phi$ and $\log M$ in the intermediate mass regime $M_1 < M < M_2$. What is the slope of this relationship?

4 points