Chemical Evolution of the Universe
Problem sheet 5

1. Consider a class of pulsating stars called Cepheids which pulsate with a period of $P \propto R/c_s$. (Note: in this problem P denotes the period and p the pressure.)
   (a) Using the main sequence scaling relations, how does $c_s \propto \sqrt{p/\rho}$ scale with the mass M and radius R?
   (b) So how does P scale?
   (c) Hence derive the period luminosity relation
   $$M = A \log P + B$$
   and determine A, where $M = -2.5 \log L + \text{const}$ is the (bolometric) absolute magnitude. Assume that the star's mass and effective temperature remain constant.

2 points

2. A main sequence star of $M = 1\,M_\odot$ was measured to have a rotation velocity of $50\,\text{km/s}$. Assume that the relation between the star's moment of inertia and its mass and radius remains constant throughout the evolution of the star.
   (a) What will be the star's rotational velocity and period once it has evolved into a red giant with $R = 100\,R_\odot$?
   (b) Let's assume that during the red giant phase the star loses half of its mass, carrying away half of its angular momentum. What will be the rotational velocity and period of the final white dwarf?

2 points

3. A gas is said to be degenerate when its degeneracy pressure exceeds its classical pressure. The electron degeneracy pressure of a fully ionised mono-atomic gas is given by
   $$P_{d,e} = \frac{1}{20}\left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{m_e} \left(\frac{Z}{A\,m_u}\right)^{5/3} \rho^{5/3}$$
   (a) Derive an expression for the critical temperature T_c below which fully ionised helium gas becomes degenerate.
   (b) What is therefore the maximum temperature of a Helium white dwarf with $M = 1\,M_\odot$?

2 points

4. Consider a core-collapse supernova in which the degenerate core of $M_c = 1\,M_\odot$ collapses to a radius of just $15\,\text{km}$. Observations showed that at its peak it had a maximum (bolomometric) absolute magnitude of $M_{\max} = -20.5\,\text{mag}$. Afterwards, its luminosity decreased by $0.15\,\text{mag}$ per day (i.e. its absolute magnitude increased by this amount). Additional observations showed that the supernova expelled material of total mass $M_e = 1\,M_\odot$ at a velocity of $10\,000\,\text{km/s}$.
   (a) How much energy is liberated by the collapse of the core?
   (b) How much energy is radiated away during the first 100 days following the explosion?
   (c) How much energy was carried away by the ejecta?
   (d) What about the rest of the energy? Where did it go?

   Hint: The bolometric absolute magnitude of the Sun is $M_{b,\odot} = 4.76$ mag.

4 points