Chemical Evolution of the Universe
Problem sheet 4

1. Consider a star of mass M and radius R. We will now estimate its central temperature on the basis of a very simple assumption.
   (a) Write down the star's radial mass density profile, $\rho(r)$, assuming that it varies linearly from ρ_0 at its centre to 0 at its surface.
   (b) Use the mass distribution equation (1st differential equation) from the lecture to compute the enclosed mass at radius r, $M(r)$.
   (c) Express ρ_0 in terms of M and R.
   (d) Use the hydrostatic equilibrium equation (2nd differential equation) from the lecture to compute the radial pressure profile, $p(r)$. Remember that the pressure is 0 at the surface of the star.
   (e) Now assume that the equation of state of the interior of the star is that of an ideal gas, i.e. $p = \frac{k}{\mu\, m_H}\,\rho T$, where μ is the mean particle mass in atomic mass units, and thus write down $T(r)$.
   (f) What is the temperature at the centre of the star, T_0?
   (g) Assuming that the star is made only of H and He, with a He mass fraction of $Y = 0.25$, what is the value of μ?
   Hint: recall problem 1. on problem sheet 3.
   (h) Finally, calculate the central temperature of the Sun.

5 points

2. Consider a star of mass $M = 10\, M_\odot$ that consists only of H at first. Let's assume that in its core 20% of its mass are converted entirely into ^{56}Fe.
   (a) How much energy is produced in total?
   (b) What fraction of this energy is produced in the first stage of the conversion process, i.e. by fusing H into He?
   (c) How much time does this star therefore spend on the main sequence?

   Hint 1: The mass numbers A are (in atomic mass units $u = 1.66054 \times 10^{-27}$ kg): 1.00728 for H, 4.00151 for ^{4}He and 55.92068 for ^{56}Fe.
   Hint 2: The lecture notes contain a scaling relation between M and luminosity L for main sequence stars.

3 points

3. Consider a spherical gas cloud with mass M, radius R and a constant density profile that is in the process of becoming a star, i.e. a pre-main-sequence star in which nucleosynthesis has not yet set in. Let's assume that it is not quite in hydrostatic equilibrium such that it its pressure cannot quite balance its gravity, meaning that it contracts slowly. Let's assume further that some mechanism exists that converts the gain in potential energy caused by the contraction into radiation with some luminosity L, and that the changing potential energy of the cloud is the only energy source of L.
   (a) Derive an expression for the rate of change of R.
   (b) By how many meters per year does a pre-MS star of $1\, M_\odot$, $2\, L_\odot$ and $3\, R_\odot$ contract?
   (c) After how many years has the star contracted by a factor of 2?

2 points