Chemical Evolution of the Universe
Problem sheet 2

1. (a) In the lecture we derived a relationship between the Hubble parameter H and the temperature T for the very early Universe where the mass-energy density is dominated by relativistic particles. Using your knowledge of $R(t)$ during this period from problem sheet 1, show that

$$kT(t) = \frac{0.85\,\mathrm{MeV}}{\sqrt{t/\mathrm{s}}}$$

   (b) How does this relation change if photons are the only relativistic particles (instead of photons, e^-, e^+, ν_e and $\bar{\nu}_\mathrm{e}$, as assumed in the lecture)?

3 points

2. As we have seen in the lecture, nucleosynthesis of ^{4}He and heavier nuclei requires the existence of deuterium first (deuterium bottleneck).
   (a) Compute the binding energy of a deuterium nucleus.
   (b) What is the frequency of a photon capable of destroying a deuterium nucleus?
   (c) At what temperature does a black body emit most of its photons at the above frequency?
   (d) At what time does the Universe reach this temperature (using the relation from 1.(b))?

4 points

3. Contrary to the result above, your lecture notes will tell you that deuterium formation sets in somewhat later. We will now learn why.
   (a) The present-day density parameter of baryons is $\Omega_{\mathrm{b},0} = 0.045$. Let us assume that all baryons are protons and that $H_0 = 70\,\mathrm{km/s/Mpc}$. Calculate the present-day number density of baryons, n_b.
   (b) The present-day temperature of the Cosmic Microwave Background is $T_0 = 2.73\,\mathrm{K}$. Use the Bose-Einstein distribution to compute the present-day number density of photons, n_γ.
   Hint: $\int_0^\infty \frac{x^2}{e^x - 1} dx = 2.4041$
   (c) What is the present-day value of the baryon-to-photon ratio, $\eta = \frac{n_\mathrm{b}}{n_\gamma}$?
   (d) How does η evolve with time (qualitatively)? Specifically, at the time you found in 2.(d), is η larger or smaller than its present-day value, or equal to it? Give your reasoning.

   Let's summarise what we've found out so far. At the time in 2.(d), a typical photon has just about enough energy to dissociate a deuterium nucleus. At later times, when the Universe has further cooled, a typical photon will therefore *not* have enough energy to dissociate a deuterium nucleus. However, in 3.(d) we found out that there are *many* more photons than baryons in the Universe at this time. Thus, although a *typical* photon cannot dissociate a deuterium nucleus any longer, there are still plenty of higher energy photons that can.

   (e) At the time you found in 2.(d), compute the fraction of photons with energy larger than the binding energy of deuterium, i.e. $n_\gamma(h\nu > \mathrm{BE_D})/n_\gamma$, again using the Bose-Einstein distribution.
   Hint 1: Use Wien's approximation (why is it ok to do that?)
   Hint 2: $\int x^2 e^{-ax} dx = \frac{e^{-ax}}{a^3}[-ax(ax + 2) - 2]$
   (f) Now calculate $n_\gamma(h\nu > \mathrm{BE_D})/n_\mathrm{b}$.
   (g) Repeat the calculation for $t = 270\,\mathrm{s}$ and verify that deuterium will start forming around this time.

8 points