SoSe 2018

Chemical Evolution of the Universe
Problem sheet 1

Recall the Friedmann equation in the following general form:

$$H(z) = H_0 \left[\Omega_{\mathrm{R},0}(1+z)^4 + \Omega_{\mathrm{M},0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\Lambda,0}\right]^{1/2} = H_0 \sqrt{E(z)}$$

where we have included radiation (R), matter (M) and a cosmological constant (Λ), and $1 + z = R_0/R$.

1. Let us first consider a Universe without a cosmological constant, i.e. $\Omega_{\Lambda,0} = 0$. We will now solve the Friedmann equation for three special cases. Each time you should make the ansatz $R(t) = C\,t^{\alpha}$. Your task is to show that in each case this ansatz does indeed solve the Friedmann equation, and to derive C and α for each case.
   (a) What is $R(t)$ in the very early Universe ($z \to \infty$)?
   (b) Assume a flat Universe. What is $R(t)$ at late times, i.e. in the distant future ($z \to -1$)?
   (c) Assume an open Universe. What is $R(t)$ at late times, i.e. in the distant future ($z \to -1$)?

   6 points

2. (a) For a given mass-energy component of the Universe with equation of state parameter w, show how its density parameter Ω depends on redshift z, i.e. what is the function $\Omega(z)$?
   (b) Show how $\Omega_k = 1 - \Omega_{\mathrm{tot}}$ depends on z.
   (c) Show how Ω_{tot} depends on z.
   (d) Consider a matter-only Universe with $\Omega_{\mathrm{M},0} = 0.9$. What is $\Omega_{\mathrm{tot}}(z = 1000)$?
   (e) Discuss: what does the result above mean for the evolution of curvature?

   3 points

3. Consider a Universe with $\Omega_{\mathrm{M},0} = 0.3$ and $\Omega_{\Lambda,0} = 0.2$ ($w_\Lambda = -1$).
   (a) What values will Ω_{M} and Ω_Λ tend to in the distant future ($z \to -1$)?
   (b) What does that mean for the future evolution of Ω_k and hence of curvature? Compare your result with that of 2.(e) above.
   (c) Express the eventual value of H in terms of $\Lambda = \frac{8\pi G \rho_\Lambda}{c^2}$.
   (d) Solve the Friedmann equation for this Universe in the distant future. I.e. what will $R(t)$ eventually look like?

   2 points