Multi-Resolution Radiative Transfer

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The Radiative Transfer Equation

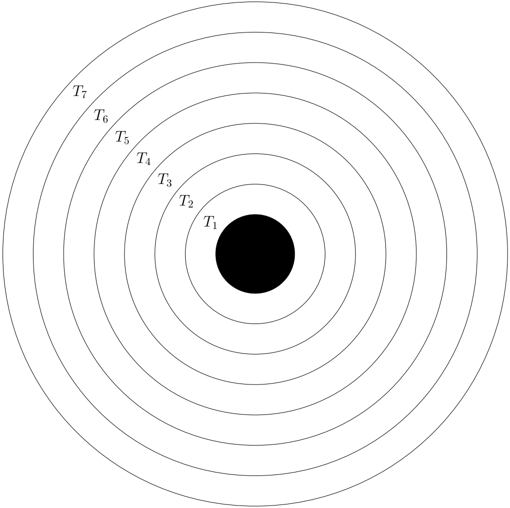
- Radiative transfer equation

$$\frac{\partial I}{\partial \tau} = I - S$$

- Formal solution

$$I_\nu(\tau_2) = I_\nu(\tau_1)\, e^{(\tau_1 - \tau_2)} + \int_{\tau_1}^{\tau_2} S_\nu(\tau)\, e^{(\tau - \tau_2)} d\tau$$

The Scattering Problem



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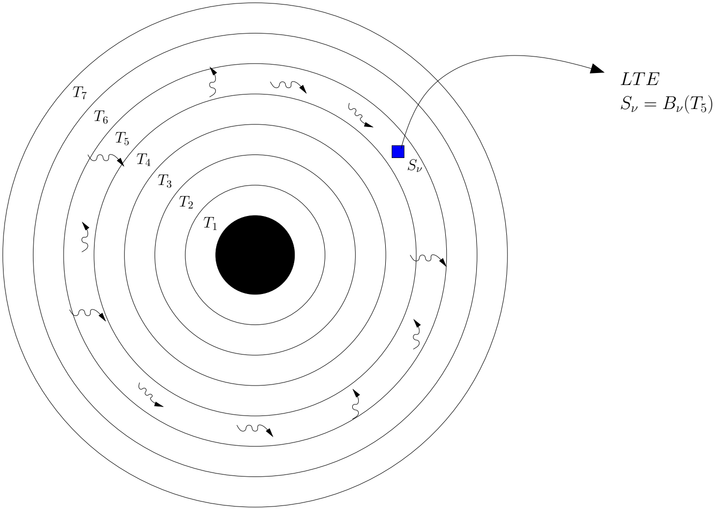

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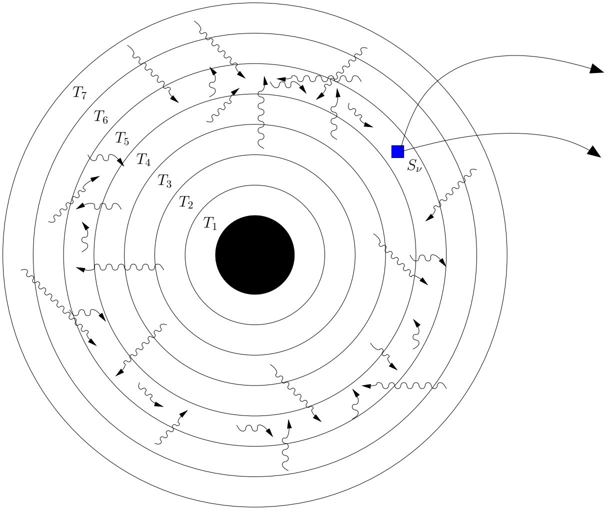


LTE

$S_\nu = B_\nu(T_5)$

NLTE

$$S_\nu = \underbrace{(1-\epsilon)J_\nu}_{\text{scattering part}} + \underbrace{\epsilon B_\nu(T_5)}_{\text{thermal part}}$$

ϵ: photon destruction probability

Basics

- static RTE, in LTE: $S_\nu = B_\nu$

$$\frac{\partial I_\nu}{\partial \tau_\nu} = I_\nu - S_\nu$$

- 1D formal solution

$$I(\tau_i) = I(\tau_{i-1})\ \exp\left(\tau_{i-1} - \tau_i\right)\ + \int_{\tau_{i-1}}^{\tau_i} S(\tau)\ e^{(\tau-\tau_i)} d\tau$$

- with NLTE source function!

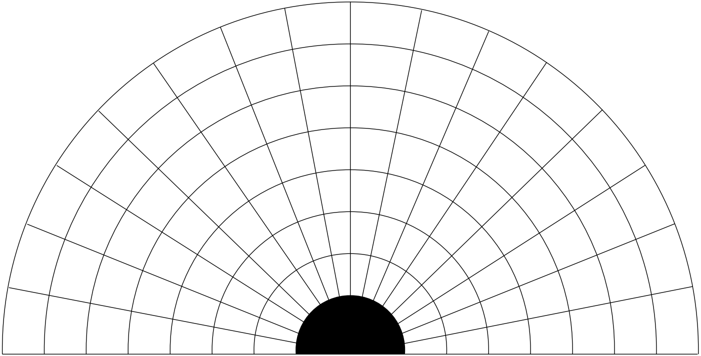
$$S_\nu = (1-\epsilon)J_\nu + \epsilon B_\nu$$

$$\text{and} \quad J_\nu(\vec{x}, t) = \frac{1}{4\pi} \oint_{4\pi} I_\nu(\vec{x}, \vec{n}, t)\ d\omega$$

- the problem is in general non-local $\rightarrow$ requires iteration

$$\frac{\partial I_\nu}{\partial \tau_\nu} = I_\nu - \left[(1-\epsilon)\left(\frac{1}{4\pi}\oint_{4\pi} I_\nu\ d\omega\right) + \epsilon B_\nu\right]$$

Radiative Transfer in PHOENIX



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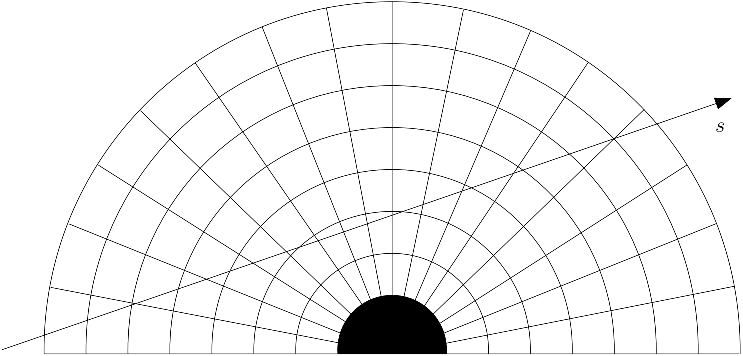

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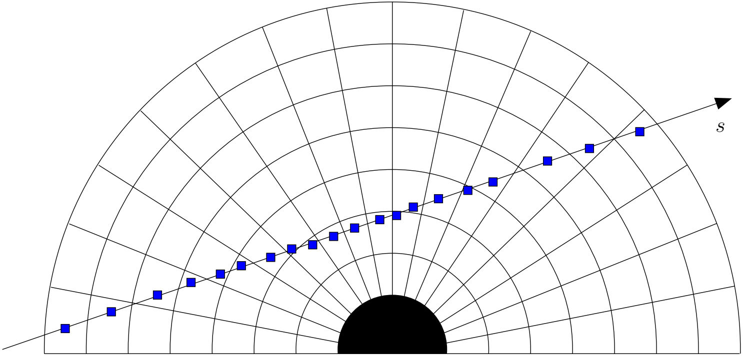

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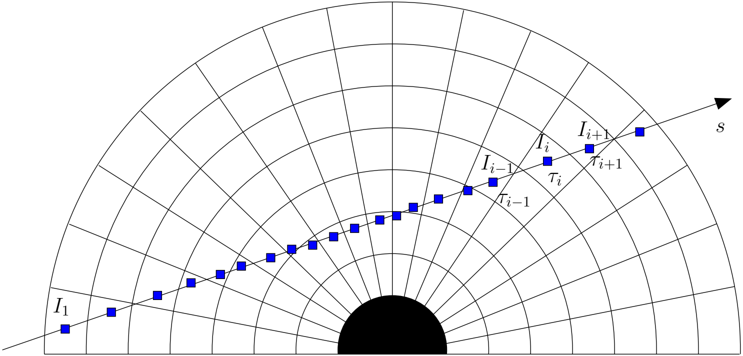

- the formal solution

$$I(\tau_i) = I(\tau_{i-1})\ \exp(\tau_{i-1} - \tau_i)\ + \int_{\tau_{i-1}}^{\tau_i} S(\tau)\, e^{(\tau-\tau_i)} d\tau$$

$$= I_{i-1}\ \exp(-\Delta\tau_{i-1}) \qquad + \alpha_i\, S_{i-1} + \beta_i\, S_i + \gamma_i\, S_{i+1}$$

The Hybrid Characteristics Method in FLASH

- combines short and long characteristics
- works on adaptive mesh and point sources
- calculates column densities

  $\Delta N = \Sigma_{\text{cells}}\ x(\text{HI})\ n(\text{H})\ \Delta s$

- no diffuse radiation

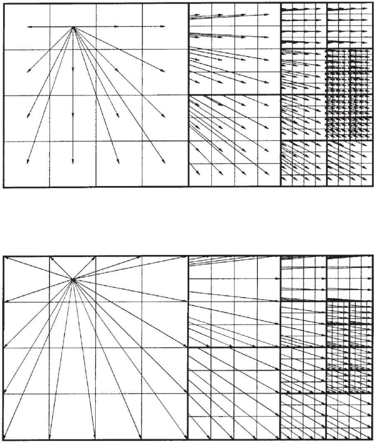

Figure: Rijkhorst 2006

Formal Solution with Hybrid Characteristics

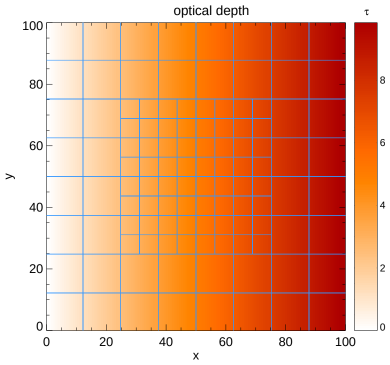

Figure: optical depth

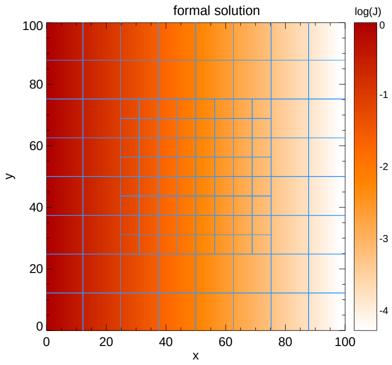

Figure: formal solution

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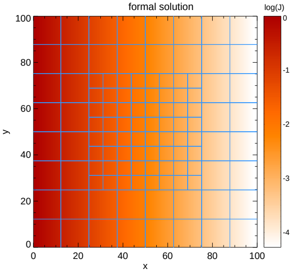

Figure: along x-axis

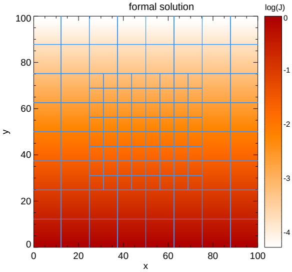

Figure: along y-axis

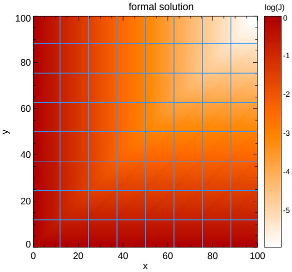

Figure: diagonal

Short Characteristics Solver in ATHENA

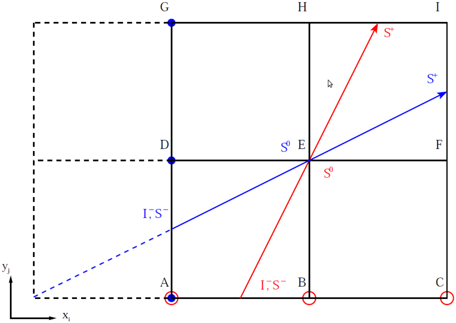

Figure: Davis, Stone, Jiang, 2012

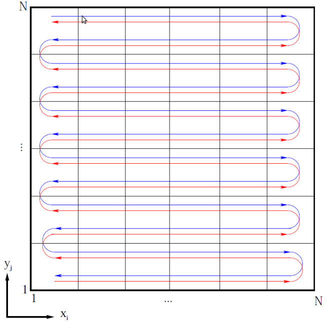

Figure: Davis, Stone, Jiang, 2012

- the formal solution

$$I_i = I_{i-1}\ \exp\left(-\Delta\tau_{i-1}\right) \qquad + \ \alpha_i\ S_- \ + \ \beta_i\ S_0 \ + \gamma_i\ S_+$$

Overview

3 steps

- calculate formal solution (with LC or SC methods)
- do an iteration step (including operator-splitting)
- solve for the new source function

Iteration

- the whole solution process is formally written with the Λ-operator

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$$J^{\text{new}} = \Lambda[S^{\text{old}}], \qquad S^{\text{new}} = (1-\epsilon)J^{\text{new}} + \epsilon B$$

- convergence rate is dramatically improved by splitting the operator

$$\Lambda = \Lambda^* + (\Lambda - \Lambda^*)$$

$$\Rightarrow J^{\text{new}} = \Lambda^*\left[S^{\text{new}}\right] + (\Lambda - \Lambda^*)\left[S^{\text{old}}\right]$$

$$\Rightarrow \Delta S = \left[1 - (1-\epsilon)\Lambda^*\right]^{-1}\left[(1-\epsilon)J^{\text{old}} + \epsilon B - S^{\text{old}}\right]$$

The Radiative Transfer Equation

- Time- and wavelength dependent radiative transfer equation

$$\frac{\partial I}{\partial \tau} + \frac{\partial \lambda I}{\partial \lambda} + \frac{\partial I}{\partial t} = I - S$$