Difference imaging in radio interferometry

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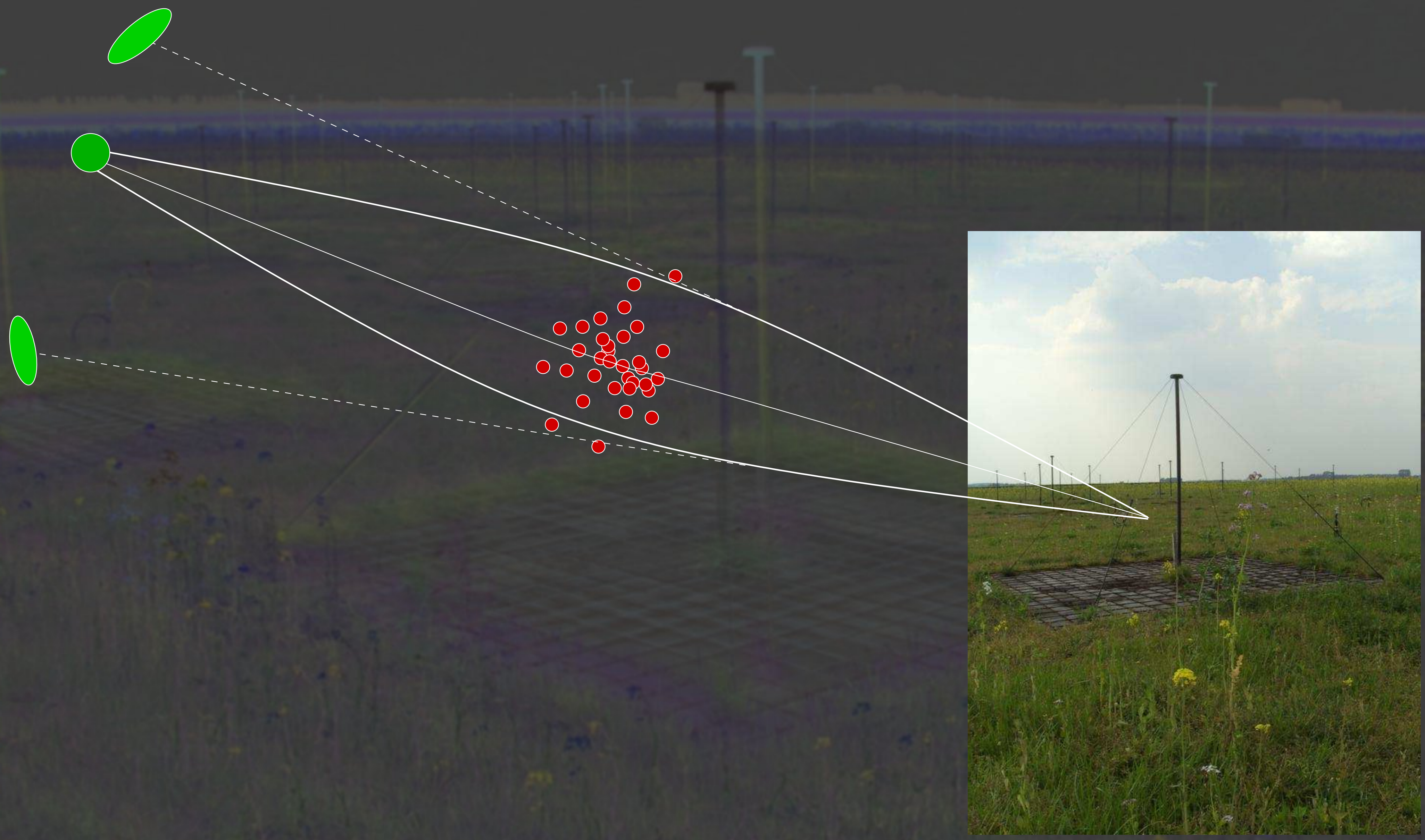
Difference imaging in radio interferometry

- Why lensing?
- Why LOFAR?
- LOFAR (lens) surveys
- Use variability
- Difference imaging

Strong gravitational lensing



Motivation for modelling

- Hubble constant

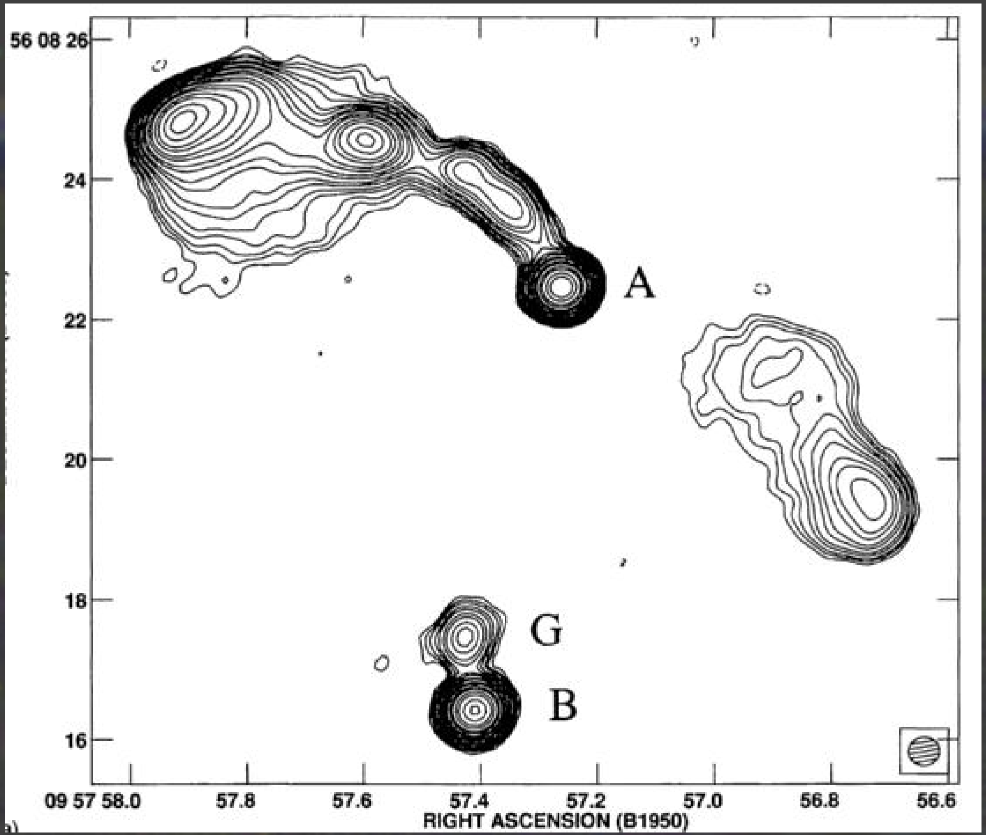
- direct information about mass distribution

  - ⋆ luminous and dark
  - ⋆ even high redshift
  - ⋆ un-biased by light

⇝ unique tool for structure and evolution of galaxies

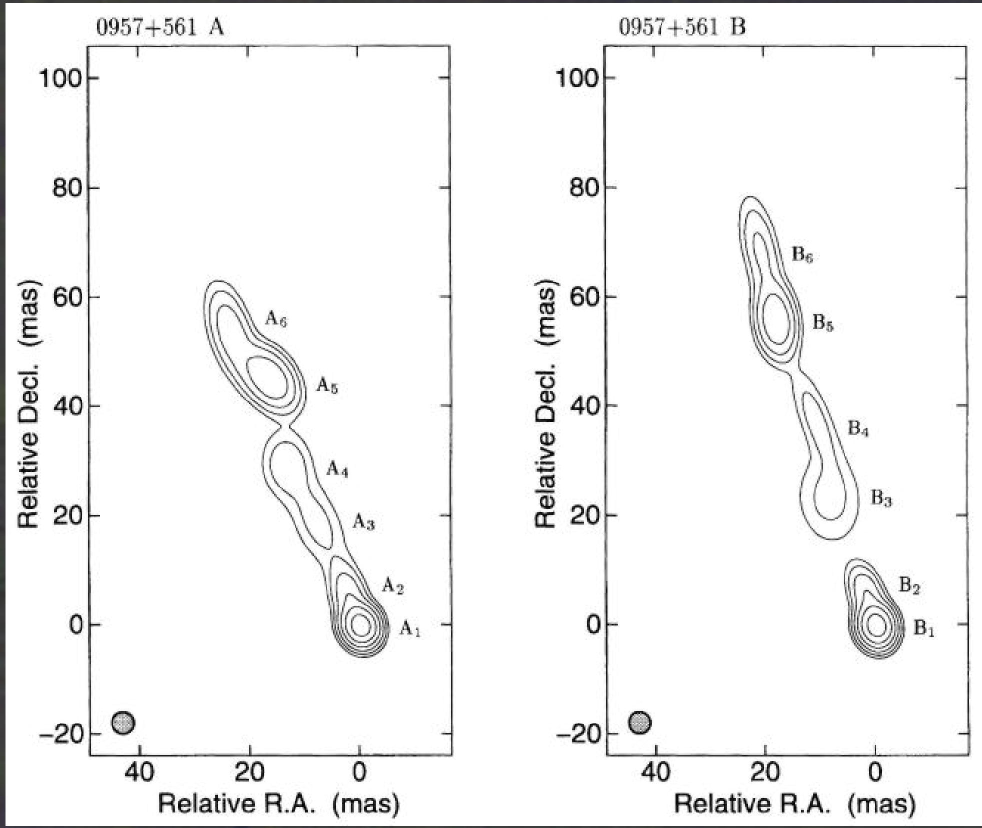
  - ⋆ large-scale mass profile
  - ⋆ CDM substructure
  - ⋆ central mass concentrations (central images)

Mass model constraints for 0957+561

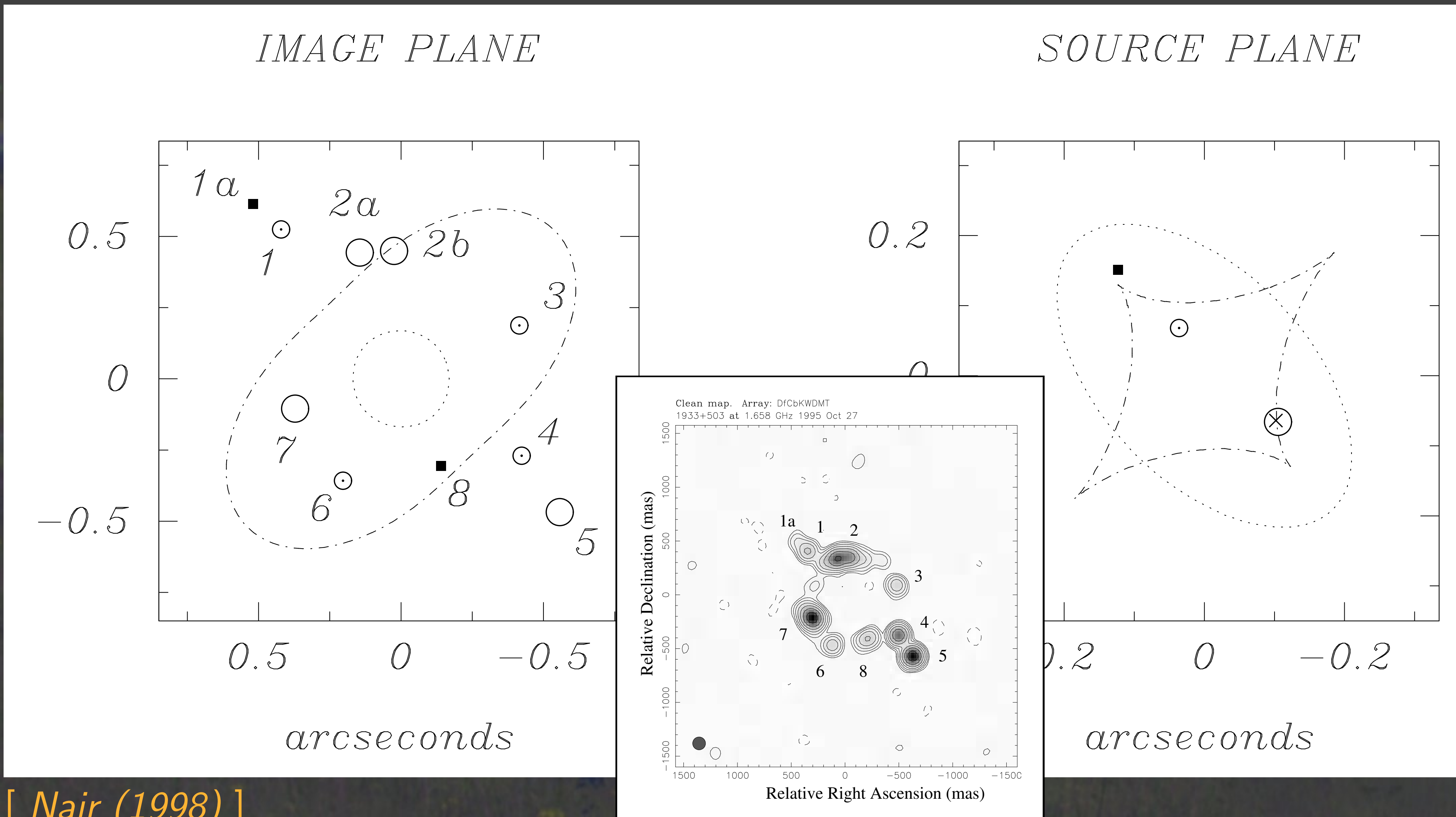


VLA [*Harvanek et al. (1997)*]

VLBI [*Garrett et al. (1994)*]



The ten image system B1933+503

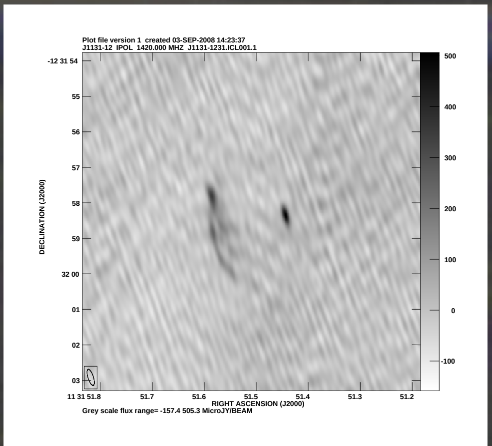


[Nair (1998)]

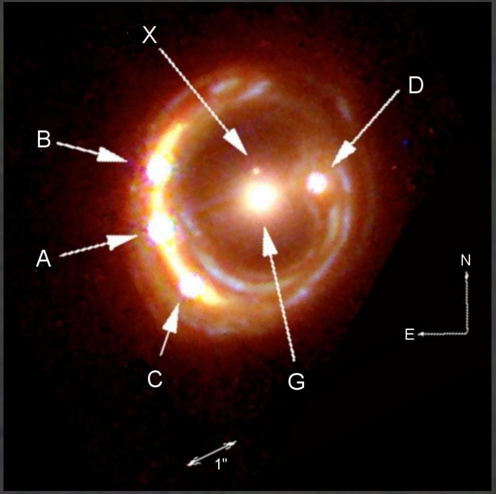
[Sykes et al. (1998)]

Extended sources

- good: compact sources provide some information
  - ⋆ CLASS survey
- better: many components
  - ⋆ not many systems
- best: lensed extended sources
  - ⋆ find them!
- go to low frequencies
  - ⋆ low surface brightness
  - ⋆ steep spectrum



[Wucknitz, in prep.]



[Claeskens et al. (2006)]

Potential for LOFAR lens searches

- 400 km baselines with sufficient sensitivity
- lensing rate $\sim$ 1:2000 (higher for extended sources)
- direct identification possible for first time
- LOFAR-200 — 15 000 lenses

  $S/N > 30$ — 1500 lenses

  separations $>1''$ — 900 lenses

- *resolution barely sufficient*

Other ways to find lenses?

- distant things are *very* far away
- variable things are small
- variability within 1 year $L \lesssim 1\,\mathrm{ly}$
- typical distances $D \gtrsim 4\,\mathrm{Gly}$
- maximum angular size $\theta = L/D \lesssim 50\,\mu\mathrm{as}$
- extrinsic variability (scintillation): only compact sources

⇝ variable things appear unresolved

(maybe not for <cm VLBI)

Extended variability

- lenses produce multiple images of sources

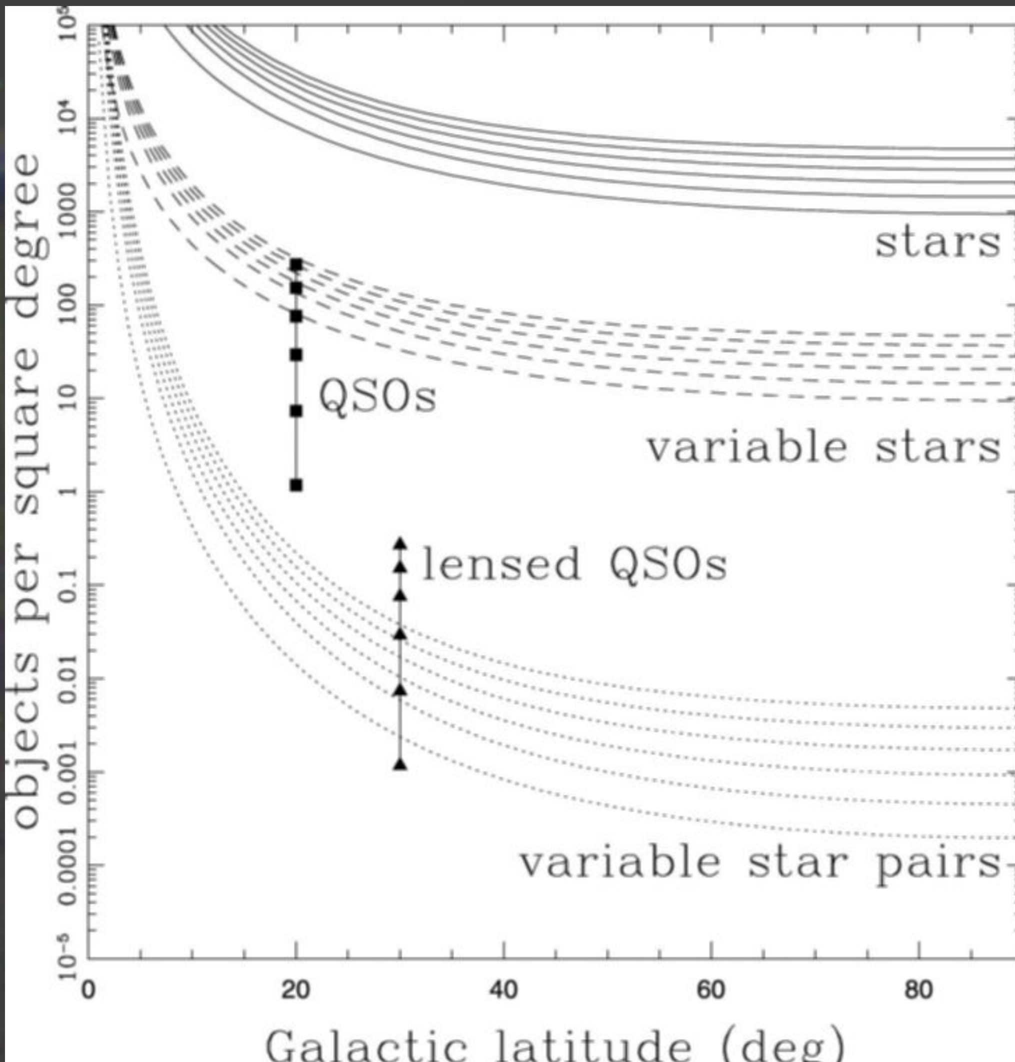
⇝ **variable part appears extended**

- expect double or quadruple variable point sources

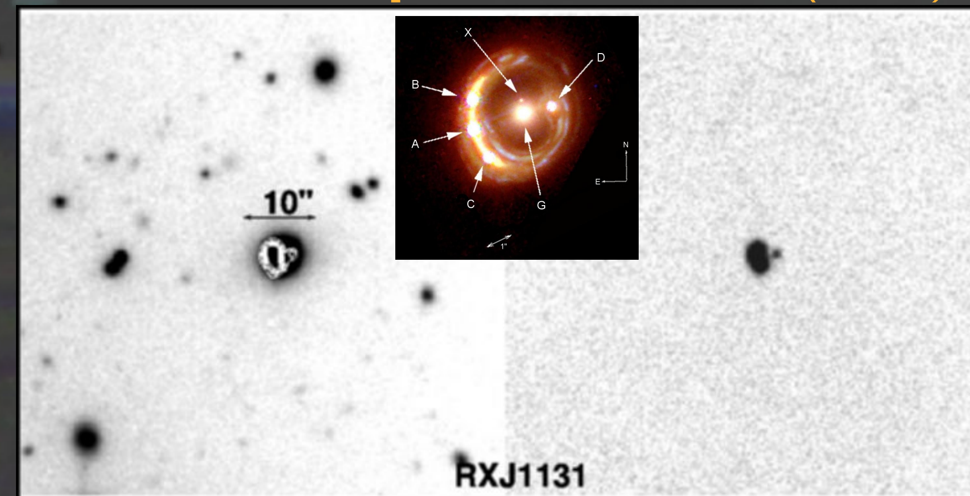
- false positives: chance alignment of multiple sources
  - ⋆ optical: variable stars, SNe
  - ⋆ radio: AGN, SNe

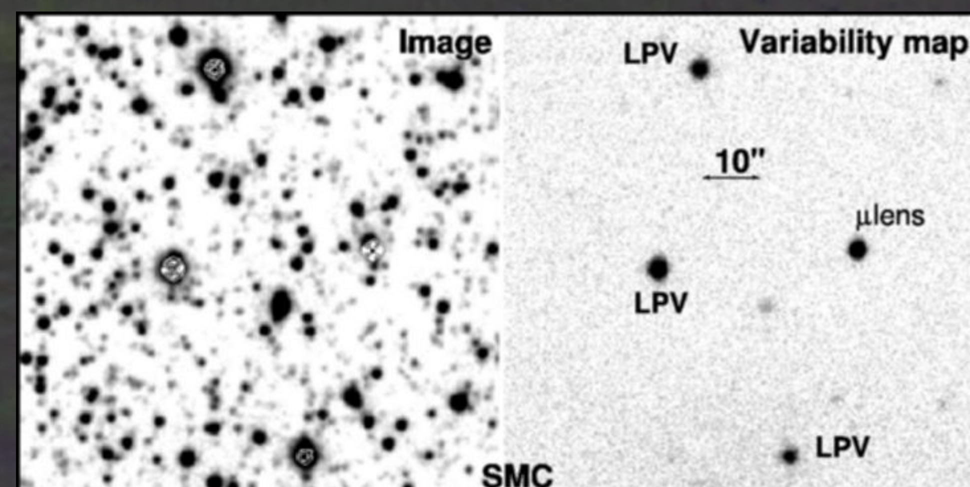
- extended variability $\neq$ extended sources with variability

Tests in the optical



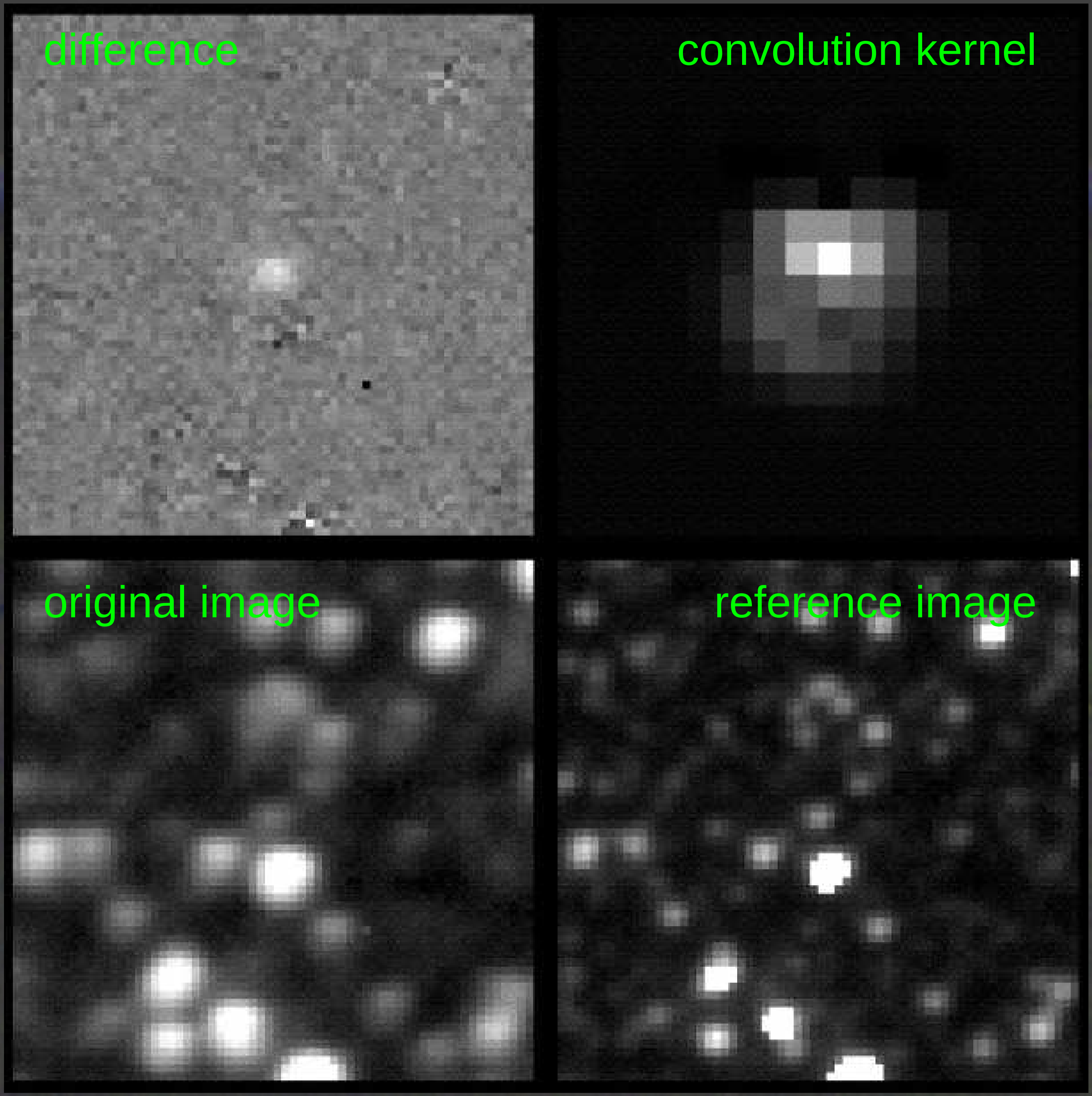
[*Kochanek et al. (2006)*]





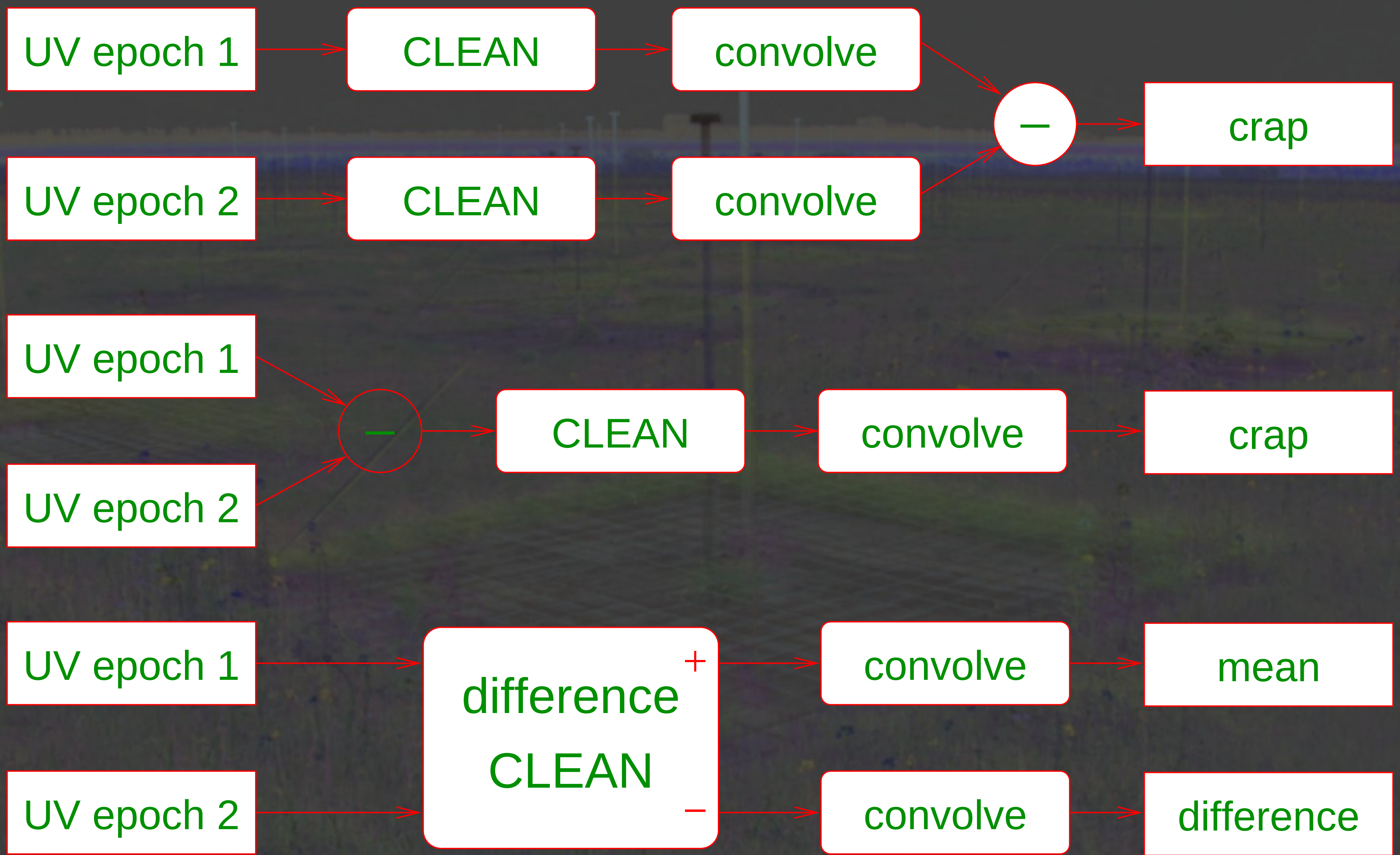
- small field in SDSS: one candidate, no lens [*Lacki et al. (2008)*]

Optical difference imaging

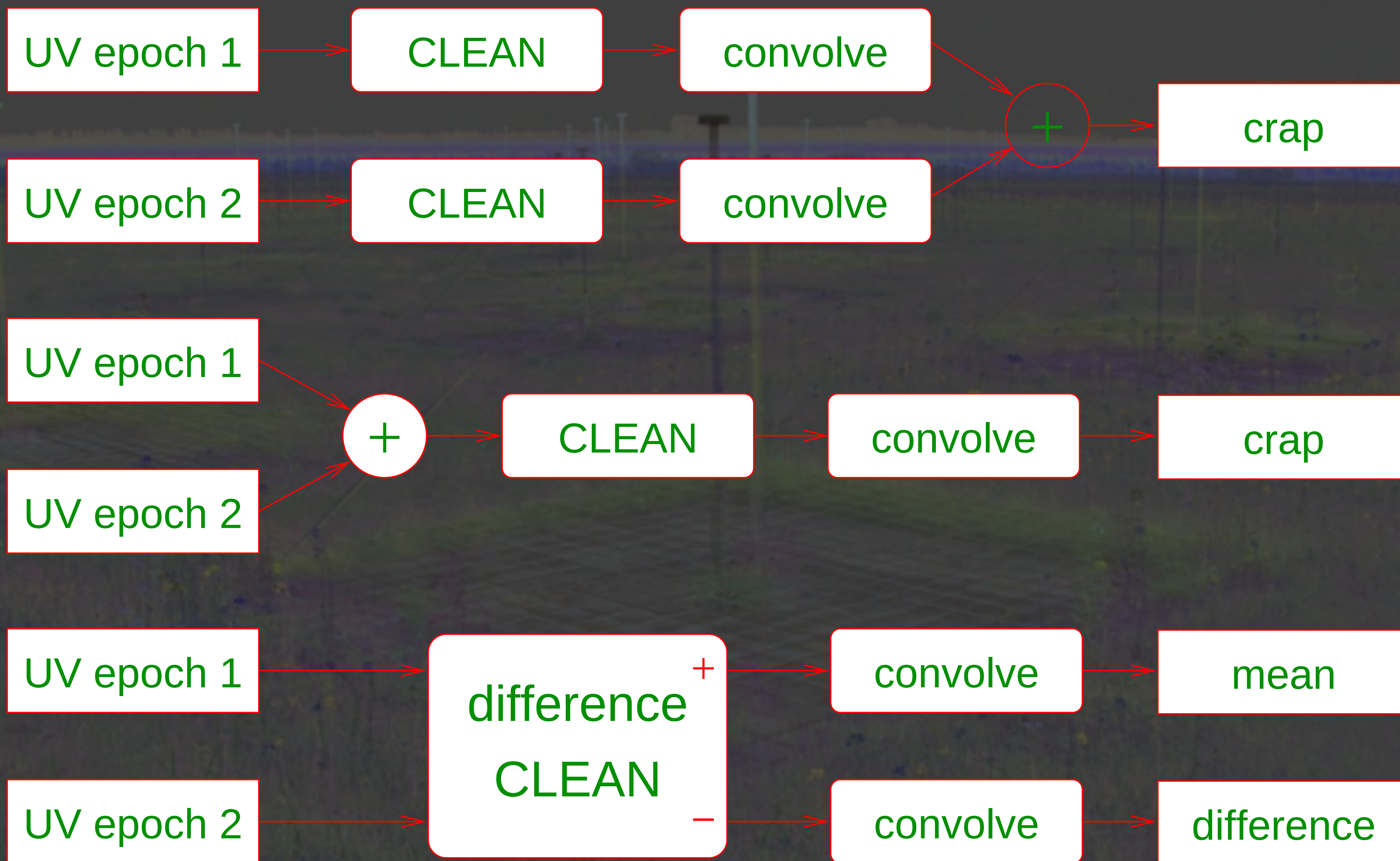


[*Alard & Lupton (1998)*]

Radio difference imaging



Related problem: combining different arrays



Notation

- sky brightness distribution $I(l,m)$ $\mathbf{I}$
- visibilities $\tilde{I}(u,v)$ $\tilde{\mathbf{I}}$
- perfect measurement
  - ⋆ Fourier transform $\tilde{I}_\mu = \sum_j A_{\mu j} I_j$

    $A_{\mu j} = \mathrm{e}^{2\pi \mathrm{i} (l,m)_j \cdot (u,v)_\mu}$
  - ⋆ vectors $\tilde{\mathbf{I}} = \mathsf{A}\,\mathbf{I}$
- residuals $R^2 = (\mathsf{A}\mathbf{I} - \tilde{\mathbf{I}})^\dagger \mathsf{W} (\mathsf{A}\mathbf{I} - \tilde{\mathbf{I}})$
- natural weighting $\mathsf{W} = \mathrm{diag}\,(\sigma_j^{-2})$ $R^2 = \chi^2$

Lazy interferometrists do it in image space

- expand $$R^2 = \tilde{\mathbf{I}}^\dagger \mathsf{W} \tilde{\mathbf{I}} + \mathbf{I}^\dagger \mathsf{A}^\dagger \mathsf{W} \mathsf{A} \mathbf{I} - 2\mathbf{I}^\dagger \mathsf{A}^\dagger \mathsf{W} \tilde{\mathbf{I}}$$
- define (using $w = \mathrm{Tr}\,\mathsf{W}$)
  - ⋆ dirty beam $$\mathsf{B} = \frac{\mathsf{A}^\dagger \mathsf{W} \mathsf{A}}{w}$$
  - ⋆ dirty map $$\mathbf{I}_{\mathsf{D}} = \frac{\mathsf{A}^\dagger \mathsf{W} \tilde{\mathbf{I}}}{w}$$
- residuals derived in image space

$$R^2 = \text{const} + \underbrace{w\left(\mathbf{I}^\dagger \mathsf{B} \mathbf{I} - 2\mathbf{I}^\dagger \mathbf{I}_{\mathsf{D}}\right)}_{R'^2}$$

minimum $\mathsf{B}\mathbf{I} = \mathbf{I}_{\mathsf{D}}$

CLEAN as maximum likelihood fitting

- add components to the model $\mathbf{I}$
- minimize residuals in each step
- empty model plus component I_j $\quad R'^2 = I_j^2 - 2 I_j I_{\mathrm{D}\,j}$
- optimal flux $$I_j = I_{\mathrm{D}\,j}$$
- residuals for optimal flux $$R'^2 = -I_{\mathrm{D}\,j}^{\,2}$$

⇝ optimal position: peak in dirty map

- subtract shifted beam from dirty map, start over

Simultaneous CLEANing

- two epochs $\tilde{\mathbf{I}}_1$ and $\tilde{\mathbf{I}}_2$
- two models/maps $\mathbf{I}_1$ and $\mathbf{I}_2$
- combined residuals

$$R^2 = \mathrm{const} + w_1 \left(\mathbf{I}_1^{\dagger} \mathsf{B} \mathbf{I}_1 - 2\mathbf{I}_1^{\dagger} \mathbf{I}_{\mathrm{D}1}\right) + w_2 \left(\mathbf{I}_2^{\dagger} \mathsf{B} \mathbf{I}_2 - 2\mathbf{I}_2^{\dagger} \mathbf{I}_{\mathrm{D}2}\right)$$

- next component at same position in 1 and 2
  fluxes independent

⇝ optimal fluxes $I_{\mathrm{D}1}$ and $I_{\mathrm{D}2}$

- optimal position maximum of $w_1 I_{\mathrm{D}1}^2 + w_2 I_{\mathrm{D}2}^2$

Need for difference-CLEAN

- disadvantages of simultaneous CLEANing
  - ⋆ deconvolution errors still independent
  - ⋆ no control over mean and difference

- alternative approach
  two channels $\mathbf{I}_+$ and $\mathbf{I}_-$

$$\mathbf{I}_+ = \tfrac{1}{2}(\mathbf{I}_1 + \mathbf{I}_2)$$
$$\mathbf{I}_- = \tfrac{1}{2}(\mathbf{I}_1 - \mathbf{I}_2)$$

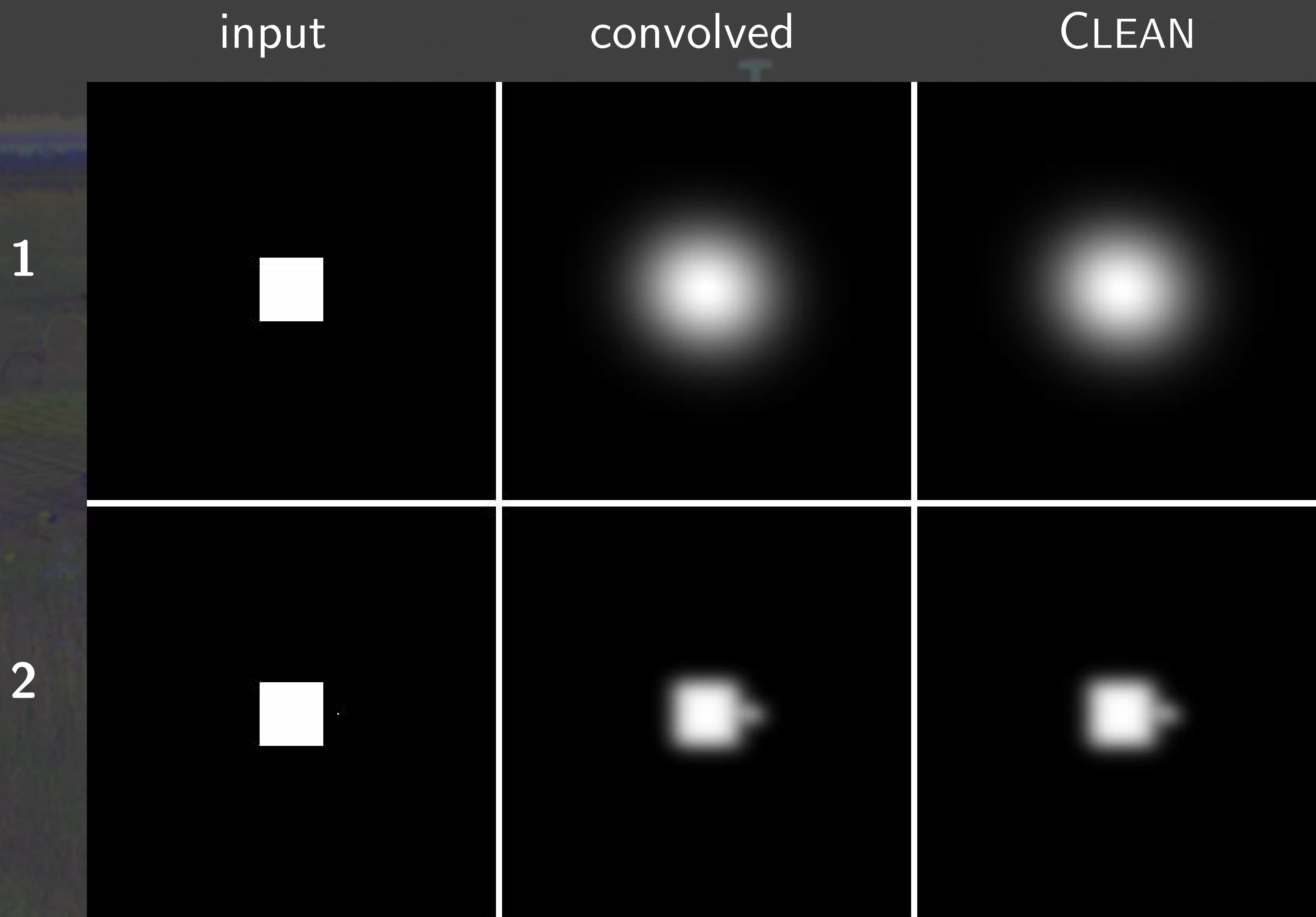
- dirty maps

$$\mathbf{I}_{\mathrm{D}+} = \frac{w_1\,\mathbf{I}_{\mathrm{D}1} + w_2\,\mathbf{I}_{\mathrm{D}2}}{w_1 + w_2}$$
$$\mathbf{I}_{\mathrm{D}-} = \frac{w_1\,\mathbf{I}_{\mathrm{D}1} - w_2\,\mathbf{I}_{\mathrm{D}2}}{w_1 + w_2}$$

D-CLEAN procedure

- next component in either $\mathbf{I}_+$ or $\mathbf{I}_-$
- residuals $$R^2 = \text{const} - (w_1 + w_2) \times \begin{cases} {I_\text{D}}_+^2 & \text{for } + \\ {I_\text{D}}_-^2 & \text{for } - \end{cases}$$
- subtract according to $$\begin{pmatrix} \mathbf{I}_{\text{D}+} \\ \mathbf{I}_{\text{D}-} \end{pmatrix} = \begin{pmatrix} \mathsf{B}_\parallel & \mathsf{B}_\times \\ \mathsf{B}_\times & \mathsf{B}_\parallel \end{pmatrix} \begin{pmatrix} \mathbf{I}_+ \\ \mathbf{I}_- \end{pmatrix}$$
- $\mathbf{I}_+$ and $\mathbf{I}_-$ not independent
- dirty beams $$\mathsf{B}_\parallel = \frac{w_1 \mathsf{B}_1 + w_2 \mathsf{B}_2}{w_1 + w_2} \qquad \mathsf{B}_\times = \frac{w_1 \mathsf{B}_1 - w_2 \mathsf{B}_2}{w_1 + w_2}$$

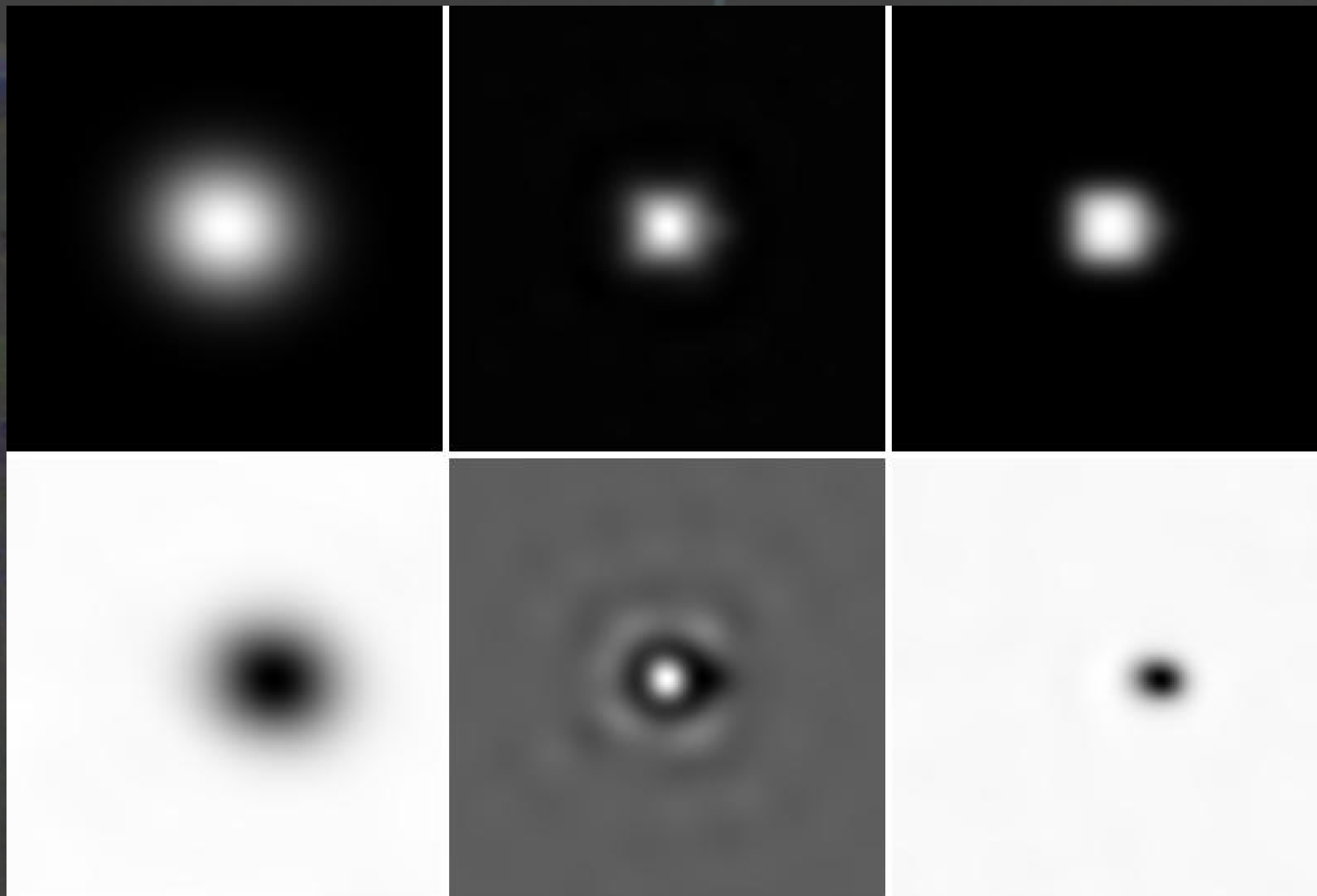
An experiment: VLA-like uv coverage, scale 1:4



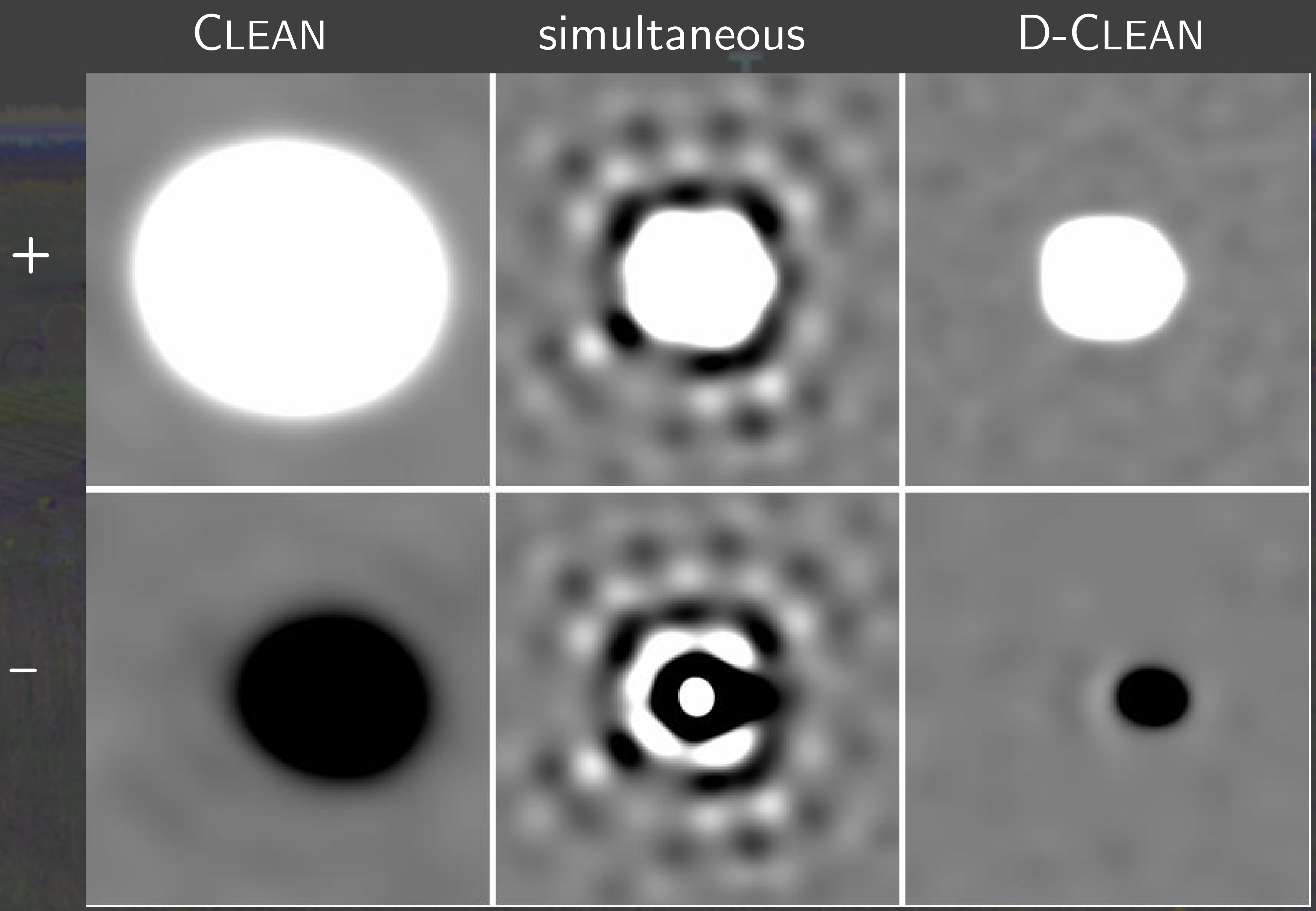
Alternative methods

CLEAN — simultaneous — D-CLEAN

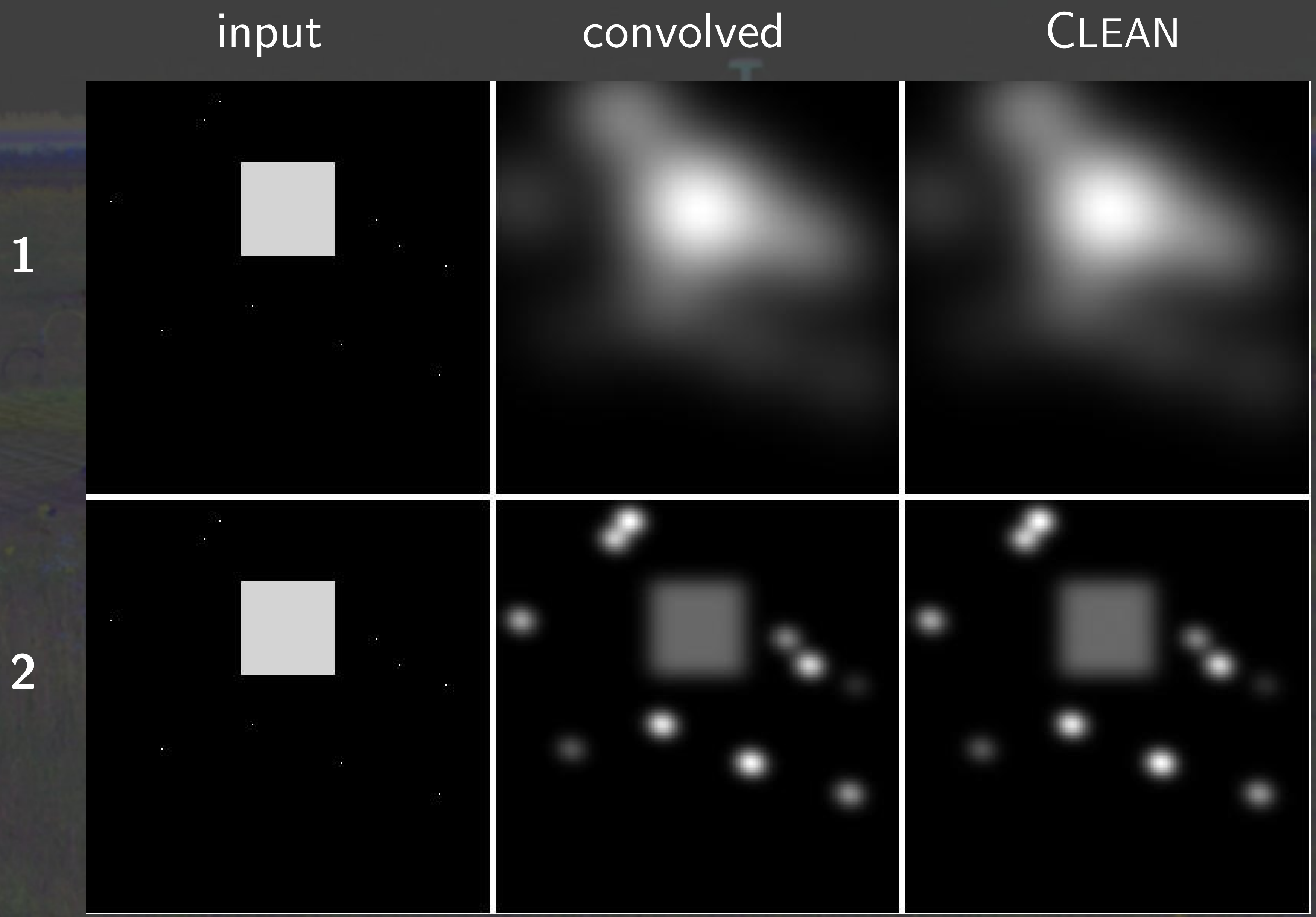
+ / −



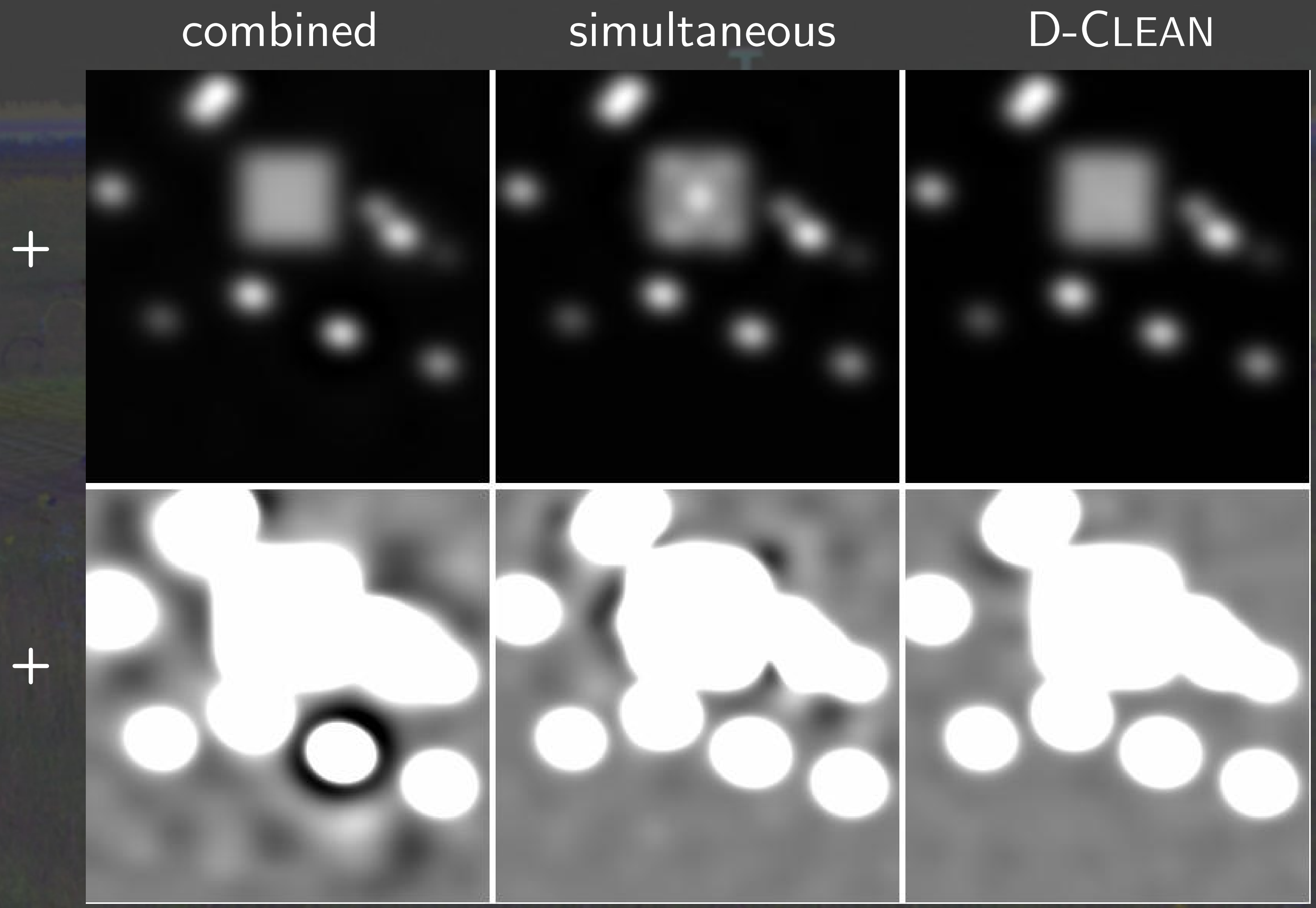
Alternative methods: residual errors



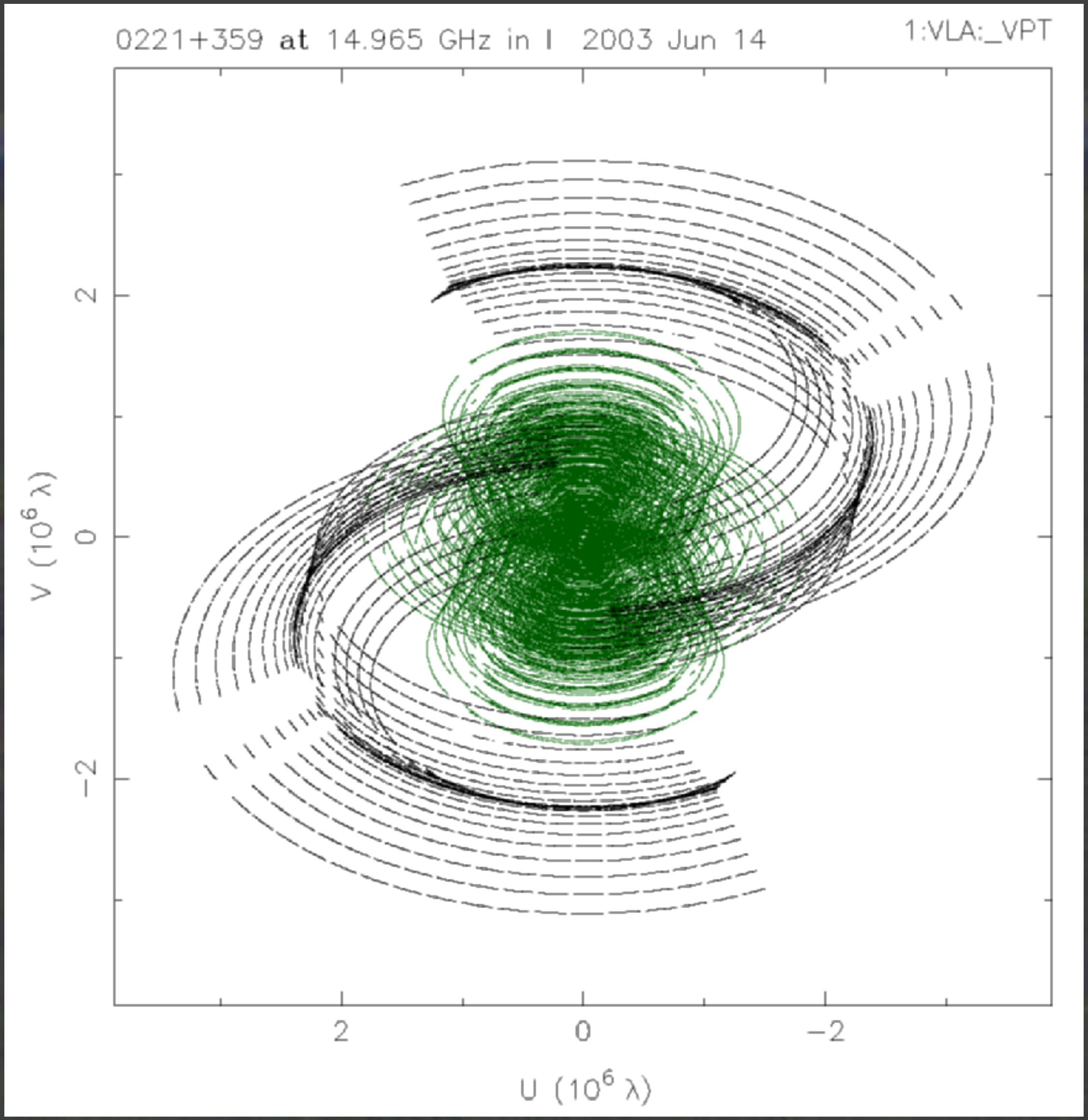
Combining different arrays: input



Combining different arrays: output



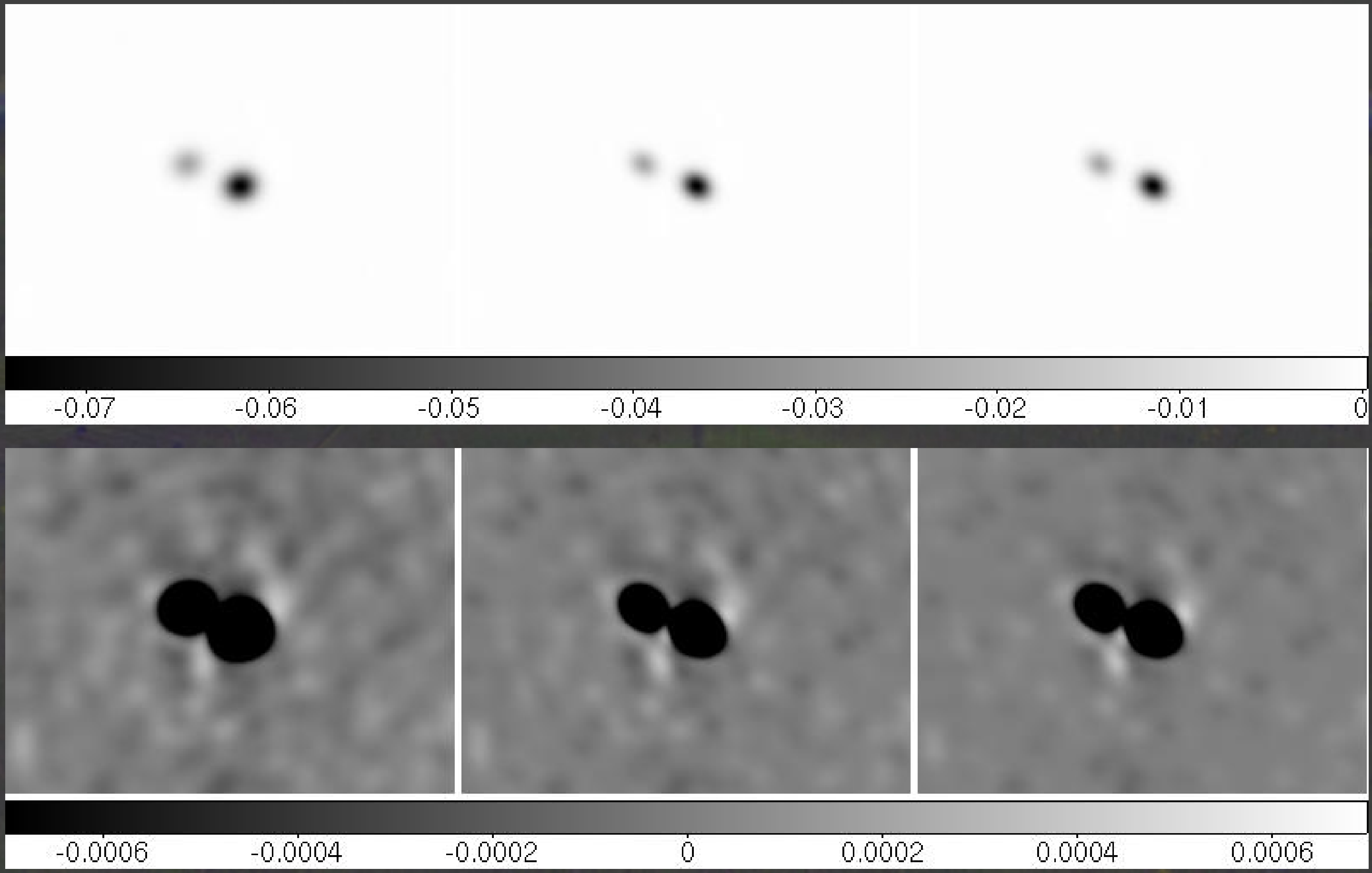
Two-channel CLEANing of real data



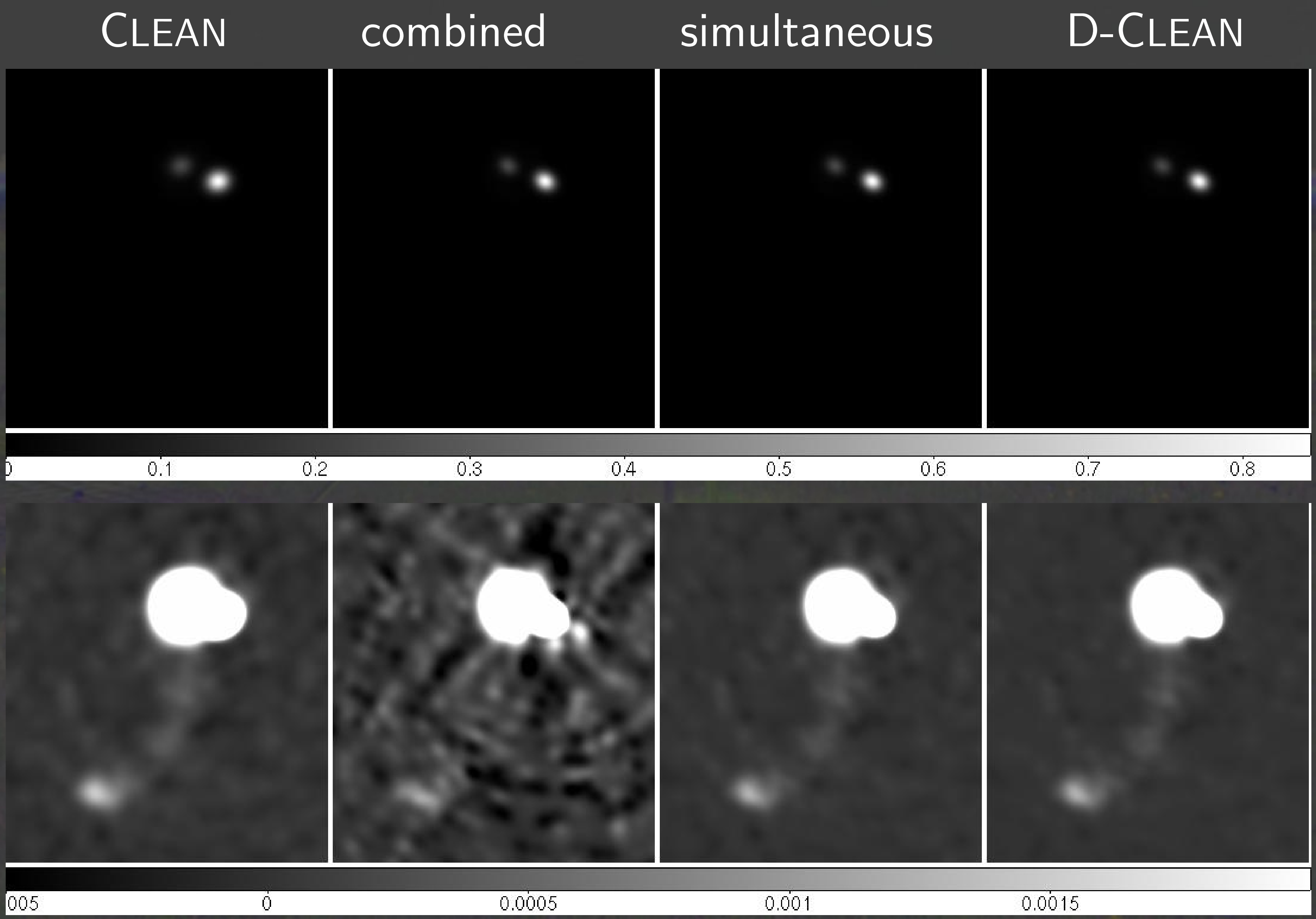
- target: lens B0218+357
  - ⋆ two bright images
  - ⋆ Einstein ring
- two VLA-A observations
  - ⋆ 1992
  - ⋆ 2003 with Pie Town
- use 1 IF

Differencing images of B0218+357

CLEAN simultaneous D-CLEAN



Combined images of B0218+357



Conclusions

- lenses are interesting
- they can be found with LOFAR
- variability is a valuable diagnostic
- developing new methods based on CLEAN
  - ⋆ difference imaging
  - ⋆ combine different arrays
- first tests encouraging
- close relation to MFS/BWS methods
- work in progress
- of interest for other applications
- speed-up with GPU

Contents